Exercise on light scattering

Exercise for week 7

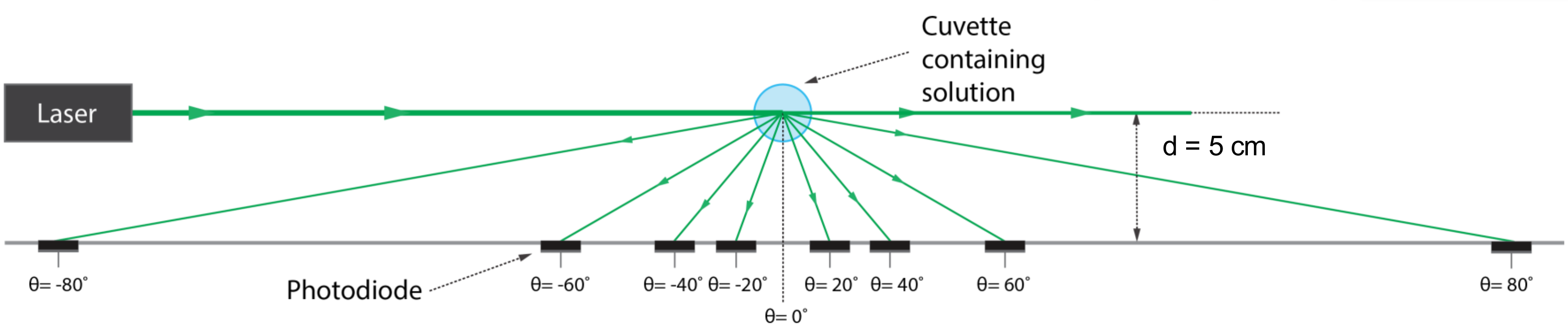
Two cylindrical cuvettes for optical spectroscopy are filled with 2 different solutions illuminated with a collimated laser beam (see the figure presented below). Flat photodetectors (photodiodes) are placed at different positions along a line that is parallel to the laser beam (line – laser beam distance: 5 cm). The surface of these photodetectors is also parallel to this beam, and they are calibrated to measure the irradiance. These photodetectors detect the scattered light coming from the solutions at various angles (see the figure presented below). The irradiances measured at different angles are given in the tables presented on the right.
The two solutions are not absorbing and they are so diluted that only single scatterings take place in the cuvettes.

Solution 1

Angle	Irradiance (mW/cm^2)
-80	0.02
-60	0.20
-40	0.60
-20	1.20
0	1.78
20	1.60
40	1.00
60	0.40
80	0.06

Solution 2

Angle	Irradiance (mW/cm^2)
-80	0.03
-60	0.30
-40	0.60
-20	1.50
0	2.07
20	1.95
40	1.80
60	2.40
80	0.60



Exercise on light scattering, Cont.

1) Plot the values of (E vs. θ) in a polar plot.

2) Plot the shape of $P(\theta)$ (probability density of finding a photon at an angle θ) in a polar plot.

3) Determine the anisotropy factor g of the two solutions.

4) By additional measurements, it was found that μ_s (solution 1) = 2.0674 cm^{-1} and that μ_s (solution 2) = 3.1662 cm^{-1}. Determine the reduced scattering coefficient μ_s' of the two solutions. What do you conclude?

Hint!

Optical properties and parameters

Interaction	Parameter		Unit	
Refraction	Refractive index*	n	[-]	ratio of the light velocity in a vacuum to its velocity in the tissue
Absorption	Absorption coefficient*	μ_a	[mm^{-1}]	probability of absorption occurring per mm
Scattering	Scattering coefficient*	μ_s	[mm^{-1}]	probablity of scattering occurring per mm
Anisotropy	Anisotropy factor*	g	[-]	describes the distribution of scattering
Reduced scattering		$\mu_s' = \mu_s(1-g)$		
Effective attenuation		$\mu_{eff} = (3\ \mu_a(\mu_a + \mu_s'))^{1/2}$		

*fundamental microscopic parameters

Hint! **Scattering Anisotropy**

The proper definition of anisotropy (g) is the expectation value for $\cos\theta$:
"Effectiveness of Scattering"

$$g \equiv \frac{\int p(\hat{s}\cdot\hat{s}')\,\hat{s}\cdot\hat{s}'\,d\Omega}{\int p(\hat{s}\cdot\hat{s}')\,d\Omega}$$

If p only depends on the angle between input and output:

$$g = <\cos\theta> = \int_0^{\pi} p(\theta)\cos\theta\cdot 2\pi\sin\theta\,d\theta \text{ where} \int_0^{\pi} p(\theta)2\pi\sin\theta\,d\theta = 1$$

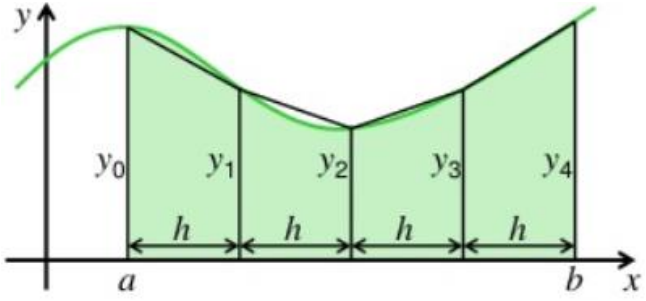
Sometimes also written in terms of $\cos\theta$: $= \int_{-1}^{1} p(\cos\theta)\cos\theta\,d(\cos\theta)$, where $\int_{-1}^{1} p(\cos\theta)d(\cos\theta) = 1$

If not, $g = \frac{\int p(\theta)\cos(\theta)\cdot 2\pi\cdot\sin(\theta)d\theta}{\int p(\theta)\cdot 2\pi\cdot\sin(\theta)d\theta}$

Hint!
Trapezoid integral approximation

$$\int_a^b f(x)dx \approx \sum_{k=1}^{N} \frac{f(x_{k-1}) + f(x_k)}{2} \Delta x_k$$

For uniform grid:



$$\Delta x_k = \Delta x = \frac{b-a}{N}$$

$$\int_a^b f(x)dx \approx \frac{\Delta x}{2} \sum_{k=1}^{N} \left(f(x_{k-1}) + f(x_k)\right)$$

$$= \frac{\Delta x}{2}\left(f(x_0) + 2\sum_{k=1}^{N-1} f(x_k) + f(x_n)\right)$$

$$= \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{N-1}) + f(x_N)\right)$$